

Morphological Image Processing

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1. INTRODUCTION

Among the major tasks in the field of image processing and analysis are feature extraction, shape description, and pattern recognition. Such tasks inherently require a geometry-oriented approach as they refer to geometrical concepts such as size, shape and orientation. However, until recently the most important tools in image processing were of a probabilistic and analytic nature, and were based upon, e.g., the correlation of signals and the frequency analysis of the Fourier spectrum.

Mathematical morphology is an approach to image processing which is based on set-theoretical, geometrical and topological concepts, and as such it is particularly useful for the analysis of geometrical structure in an image. In contrast to the traditional approach using Fourier analysis, morphology is highly nonlinear in nature, and poses several challenging mathematical problems. Below we shall briefly describe the historical development of this approach, explain some of its basic techniques, and discuss some recent theoretical developments, with an emphasis on those carried out at CWI.

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2. THE NATURE OF MATHEMATICAL MORPHOLOGY?

It is interesting to have a deeper reflection upon the origin and nature of mathematical morphology. What is it? Where does it come from? How does it operate?

The word 'morphology' stems from the Greek words *μορφή* and *λογος*

meaning ‘the study of forms’. The term is encountered in a number of scientific disciplines including biology and geography. In the context of image processing it is the name of a specific methodology designed for the analysis of the geometrical structure in an image. It was founded in the early sixties by two researchers at the Paris School of Mines in Fontainebleau, G. Matheron [3] and J. Serra [5], who worked on a number of problems in mineralogy and petrography. Their main goal was the automatic analysis of the structure of images from geological and metallurgic specimens. They were particularly interested in the quantization of the permeability of a porous medium and the petrography of iron ores. Their investigations ultimately led to a new quantitative approach in image analysis, nowadays known as mathematical morphology. During the last two decades, this discipline has gained increasing popularity among the image processing community and has achieved the status of a powerful alternative to the classical linear approach. It has been applied in numerous practical situations, e.g., mineralogy, medical diagnostics, histology, industrial inspection, computer vision and character recognition.

Mathematical morphology has three aspects: an algebraic one, dealing with image transformations derived from set-theoretical and geometrical operations, a probabilistic one, dealing with models of random sets applicable to the selection of small samples of materials, and an integral geometric one, dealing with image functionals. Only the first aspect will be addressed here.

By its very nature, mathematical morphology is set-based, that is, it treats a binary image as a set. The corresponding morphological operators use essentially only four ingredients from set theory: set intersection, union, complementation, and translation. As a result such operators are translation invariant; additionally, they are highly nonlinear.

One of the basic intuitions of mathematical morphology is that the analysis of an image does not reduce to a simple measurement. Instead, it relies on a succession of operators which transform it in order to make certain features apparent. Indeed, a picture usually contains an unstructured wealth of information; in order to analyze it, one has to distinguish meaningful information from irrelevant distortions. One has to extract what is of interest. In practice this amounts to transformations which reduce the original image to a sort of caricature. For example, in optical character recognition, one can simplify the task by first performing a *skeletonization* on a binary digital image representing a typed text, which reduces each connected component to a one-pixel-thick skeleton retaining its shape; this discards all (useless) information about the thickness of characters, and the reduced amount of information contained in such an image makes further recognition steps quicker and easier.

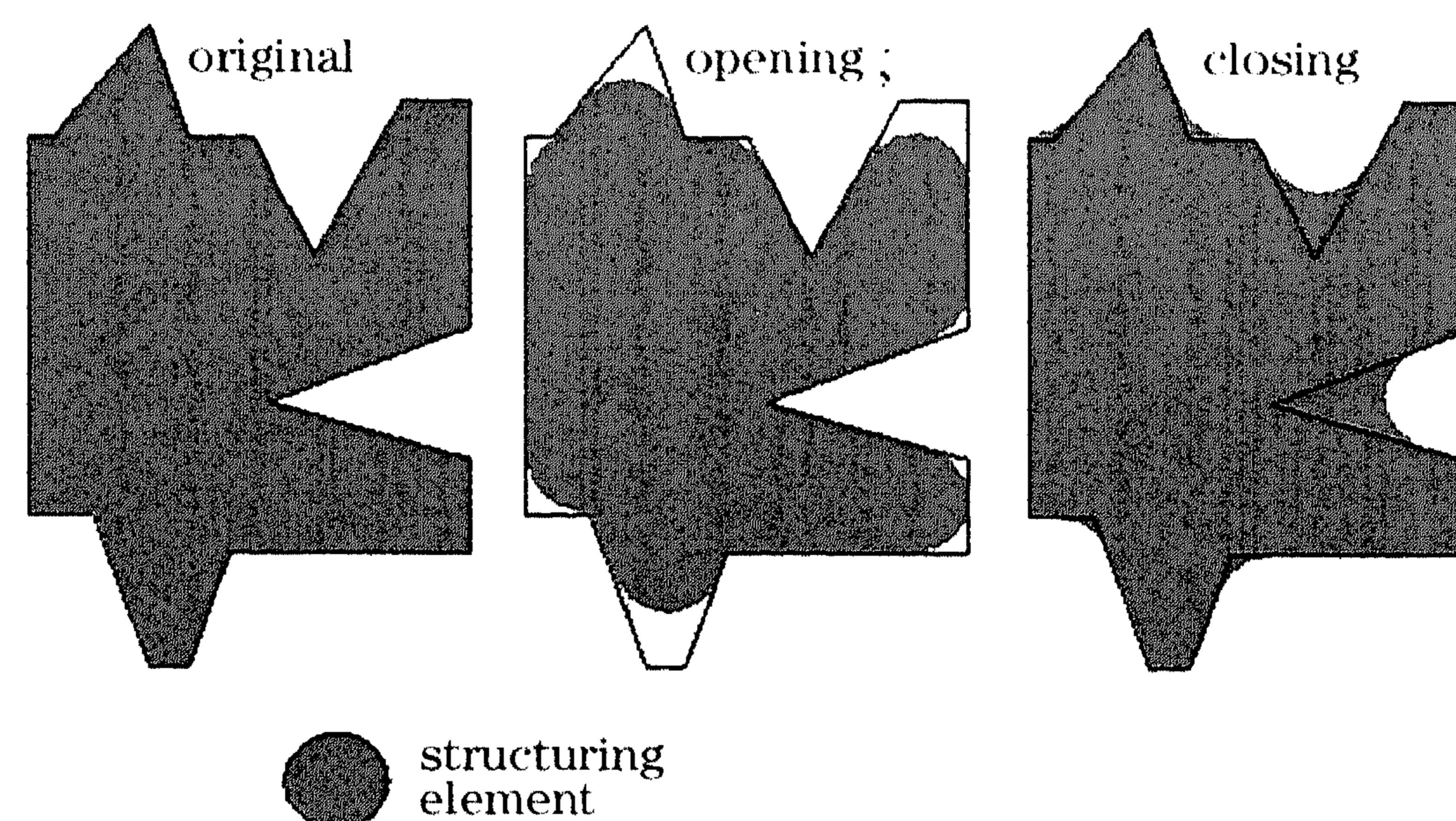


Figure 1. Opening and closing of a polygon by a disk.

3. AN EXAMPLE: OPENINGS

The central idea of mathematical morphology is to examine the geometrical structure of an image by probing it with small patterns, called *structuring elements*, at various locations in the image. By varying the size and shape of the structuring elements, one can extract useful shape information from the image. This procedure results in image operators which are well-suited for the analysis of the geometrical and topological structure of an image.

This is perhaps best illustrated by discussing one operator in more detail, the *opening*, one of the most important operators in daily morphological practice. Restricting to binary (i.e., black-and-white) images modelled by $\mathcal{P}(\mathbb{R}^2)$, the subsets of \mathbb{R}^2 , we say that the mapping $\alpha : \mathcal{P}(\mathbb{R}^2) \rightarrow \mathcal{P}(\mathbb{R}^2)$ is an *opening* if it is

- *increasing*: $X \subseteq Y$ implies $\alpha(X) \subseteq \alpha(Y)$;
- *idempotent*: $\alpha(\alpha(X)) = \alpha(X)$;
- *anti-extensive*: $\alpha(X) \subseteq X$.

We discuss three different types of openings here: the structural opening, the linear opening, and the area opening.

The *structural opening* requires a structuring element $A \subseteq \mathbb{R}^2$. It is the union of all translates of A which are contained inside X :

$$X \circ A = \bigcup \{A_h \mid h \in \mathbb{R}^2 \text{ and } A_h \subseteq X\}.$$

Here A_h denotes the translate of A along the vector h . This opening is illustrated in figure 1, along with its *negative*, the structural closing by A , which is essentially an opening of the background. A closing operator, say β , is increasing, idempotent, and extensive (i.e., $X \subseteq \beta(X)$).

The *linear opening* uses a (finite or infinite) collection of bounded line segments L_i , $i \in I$, with different directions for structuring elements. It is defined by

$$\alpha_L(X) = \bigcup_{i \in I} X \circ L_i.$$

Finally, the *area opening* uses the notion of (arc-)connected component. Let $S \geq 0$ be a real number, then $\alpha_S(X)$ comprises all components of X with area larger than S . The three different openings are illustrated in figure 2.

Openings are used for different purposes, such as image filtering (see section 5). Here we discuss a different application, the computation of size distributions.

Consider the family of structuring elements rB , the spheres in \mathbb{R}^2 centered at the origin and with radius $r > 0$. The family of structural openings $\alpha_r(X) = X \circ rB$ satisfies the following semigroup property:

$$\alpha_r \alpha_s = \alpha_s \alpha_r = \alpha_r \quad \text{if } r \geq s.$$

This is due to the fact that a larger ball can be obtained as a union of smaller ones. This semigroup property forms the basis for a formal definition of a *size distribution*. The openings α_r formalize the intuitive idea of the sieving of a binary image according to the size and shape of grains within the image. As the mesh size of the sieve (the radius r) is increased, more of the image grains will fall through the sieve and the residual area of the filtered (sieved) image will decrease monotonically. These residual areas form a size distribution, called *granulometric size distribution*, that is indicative of the image structure. Its derivative is a density function, called the *granulometric size density*.

The opening transform of a binary image $X \subseteq \mathbb{R}^2$ is a function $F : \mathbb{R}^2 \rightarrow \mathbb{R}_+$ whose value $F(h)$ at the point h represents the radius r of the largest sphere which contains h and fits entirely inside X . Its histogram corresponds with the granulometric size density. See figure 3 for an illustration.

4. COMPLETE LATTICE FRAMEWORK

Although, originally, mathematical morphology was developed for binary images, from the very beginning there was a need for a more general theory. Such a theory should be powerful enough to handle different object spaces such as the closed subsets of a topological space, the convex sets of a (topological) vector space, and grey-scale images.

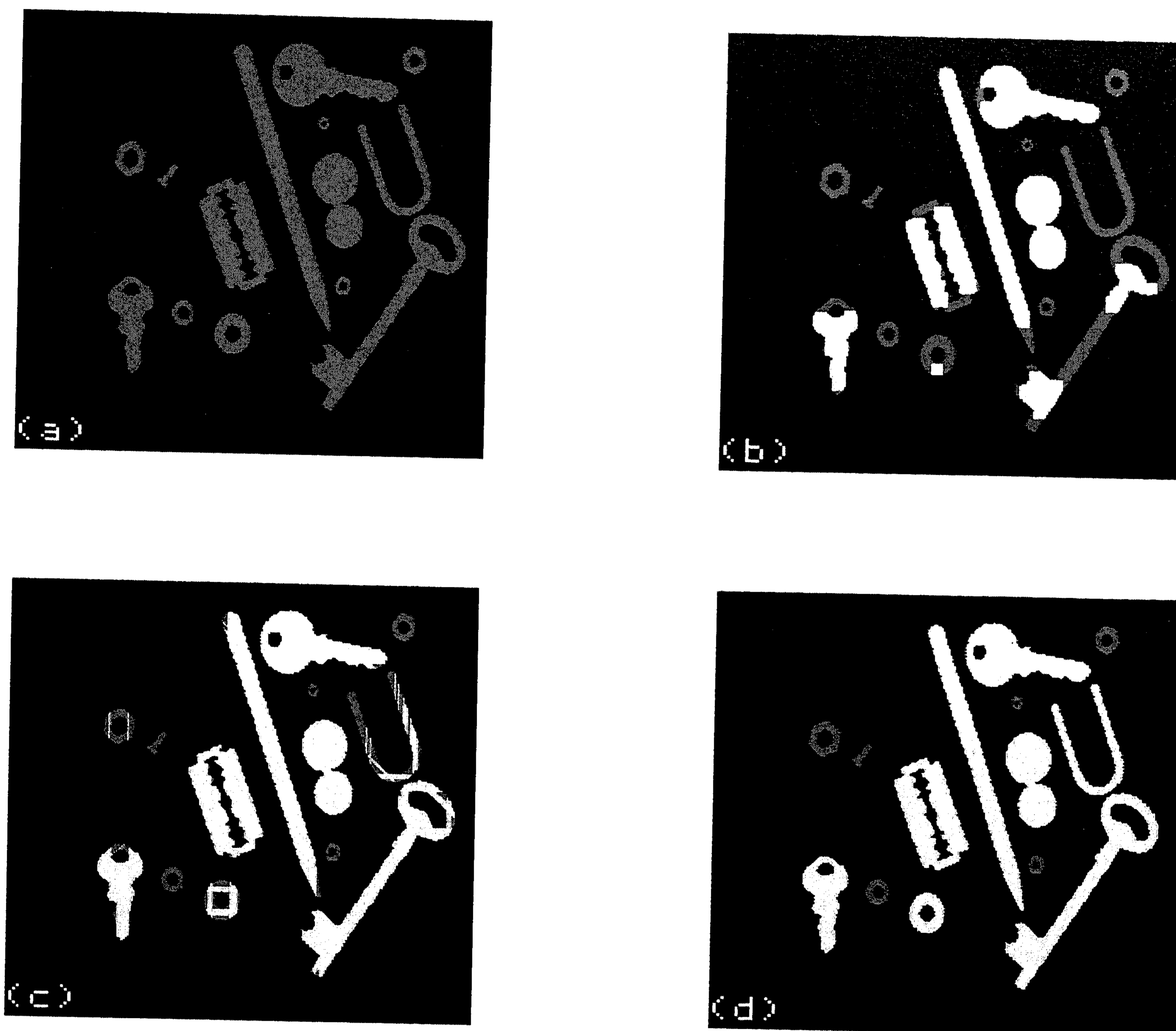


Figure 2. Three different openings. (a) binary input image (foreground in red); (b) the structural opening by a 7×7 square (yellow); (c) linear opening using four line segments (horizontal, vertical, and diagonal) with length 15; (d) area opening with $S = 256$.

Besides this enormous variation in object spaces there is yet another generalization which is quite important. It is, namely, by no means obvious why morphological operators have to be translation invariant. In radar imaging, for example, rotation invariance is more appropriate. Furthermore, there are a number of situations where perspective transformations enter naturally. Think, for instance, of the problem of monitoring the traffic on a highway with a camera at a fixed position. It is obvious that in such a configuration the detection algorithms should take into account the distance between the camera and the object (e.g. a car).

Only recently has it been realized that complete lattices are the right mathematical framework for a general theory of morphology: see [1, 2, 4,

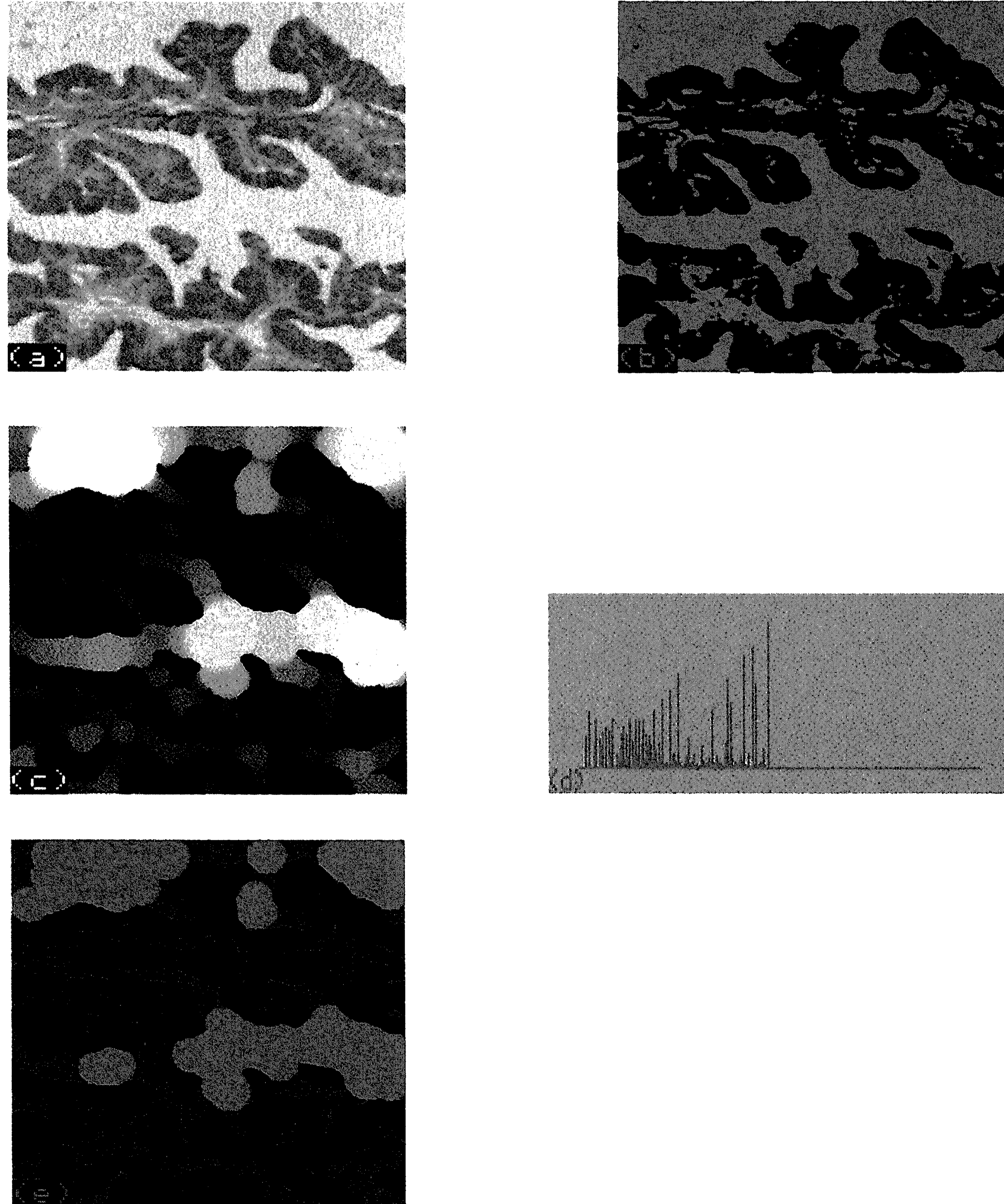


Figure 3. Opening transform and size distribution. (a) Grey-scale image; (b) binary image obtained by thresholding; (c) its opening transform; (d) the histogram of grey-values of the opening transform (which corresponds with the granulometric size density); (e) the binary image (red), obtained by thresholding the opening transform. In this particular case the 5-7-11-chamfer metric has been used as an accurate discrete approximation of the continuous Euclidean distance.

6]. The main motivation for this generalization is that it unifies a number of particular examples into one abstract mathematical framework; they help to prevent the periodic ‘reinvention of the wheel’ which happens too often in applied mathematics and engineering, where ‘new ideas’ are sometimes particular cases of ‘old ideas’ in pure mathematics. A second motivation intimately connected to the previous one is that an abstract approach provides a deeper insight into the essence of the theory (which assumptions are minimally required to have certain properties?) and links it to other, sometimes rather old, mathematical disciplines.

The mathematical morphology research group at CWI has made a substantial contribution to the development of the complete lattice framework for morphology [1].

5. MORPHOLOGICAL FILTERS

5.1. Introduction

Another class of problems dealt with at CWI concerns the construction of morphological filters. One goal of image filtering may be the enhancement of the visual quality of a distorted image. More frequently, however, its goal is to make the image more suitable for subsequent image processing tasks, such as segmentation.

In mathematical morphology a ‘filter’ is an operator which is increasing and idempotent. Idempotence seems a sensible requirement for a filtering operation as it characterizes the successive stages of a series of transformations in image analysis. Indeed, if an operation is idempotent, then there is no point in repeating it, and so we must do something else, i.e., go to another stage. Conversely, a stage must produce a clear result, and not stop halfway.

Given a filtering operator ψ which is not idempotent, one often applies it until the result does not change anymore. This corresponds to a conditional loop, such as ‘**while** ... **do** ...’; provided such a loop eventually terminates, it implements an idempotent operation. However, in general there is no guarantee of convergence. At CWI a theory has been developed which says under what sort of conditions on the operator ψ , iteration leads to idempotence. This theory covers all the interesting cases occurring in practice.

5.2. Alternating sequential filters

By means of illustration we describe one family of morphological filters in more detail, namely the alternating sequential filters based on rank-order. This class was ‘invented’ recently by the author. To define it, we need to introduce some notation and terminology. Denote by $\text{Fun}(\mathbb{Z}^2)$ the class of grey-scale functions $F : \mathbb{Z}^2 \rightarrow \{0, 1, \dots, N\}$, where N is an integer.

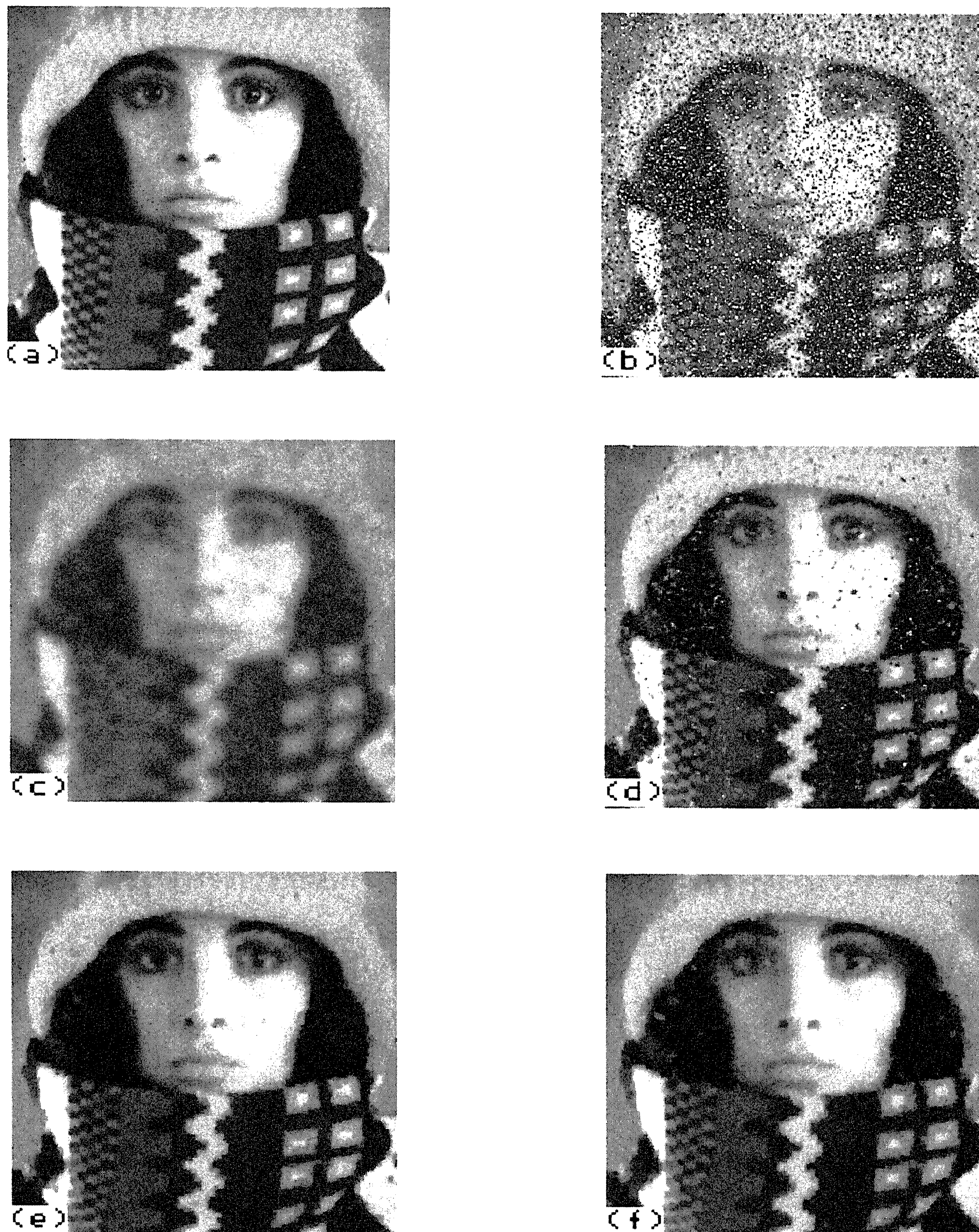


Figure 4. Morphological filtering: (a) undistorted image; (b) distorted version of (a) in which about one-third of the pixels are affected by noise; this image is used as input image F ; (c) Gaussian filtered version of F ; (d) median $\rho_5(F)$; (e) and (f) are the alternating sequential filtered images $(\beta\alpha)_9(F)$ and $(\alpha\beta)_9(F)$, respectively. Note that the linear Gaussian operator blurs the image, whereas the morphological operators are able to remove noise without blurring the image.

Let α_k , $k = 1, 2, \dots, n$, be a sequence of openings such that

$$\alpha_n \leq \alpha_{n-1} \leq \dots \leq \alpha_1.$$

For example, α_k may be the structural opening with a $(2k + 1) \times (2k + 1)$ structuring element. Dually, let β_k , $k = 1, 2, \dots, n$, be a sequence of closings such that

$$\beta_n \geq \beta_{n-1} \geq \dots \geq \beta_1.$$

Denote by $(\alpha\beta)_n$ the composition

$$(\alpha\beta)_n = \alpha_n\beta_n\alpha_{n-1}\beta_{n-1}\dots\alpha_1\beta_1.$$

The composition $(\beta\alpha)_n$ is defined analogously. Now the following result holds.

Proposition. *Under the given assumptions, the operators $(\alpha\beta)_n$ and $(\beta\alpha)_n$ are morphological filters on $\text{Fun}(\mathbb{Z}^2)$, i.e., both operators are increasing and idempotent.*

The filters $(\alpha\beta)_n$ and $(\beta\alpha)_n$ are called *alternating sequential filters*. One particular example will be discussed here. Consider the points $0, p_1, p_2, \dots, p_8$, origin; define the rank operator ρ_k (where $k = 1, 2, \dots, 9$) as follows: sort, for a given input image F and pixel $x \in \mathbb{Z}^2$, the values $F(x), F(x+p_1), F(x+p_2), \dots, F(x+p_8)$ in decreasing order and take as output $\rho_k(F)(x)$ the value at the k 'th position. The operator ρ_1 , which returns as output the maximum of the values $F(x+p_i)$, is called *dilation*, and is denoted by δ . Dually, ρ_9 , which returns the minimum is called *erosion*, and is denoted by ε . The operator ρ_5 is called the *median operator*. It is evident that

$$\rho_1 \geq \rho_2 \geq \dots \geq \rho_9.$$

The operator $\alpha_k = \text{id} \wedge \delta\rho_k$, where id is the identity operator ($\text{id}(F) = F$) and \wedge denotes the (pointwise) minimum, is an opening, called *rank-max opening*: see [4] or [1]. It follows that α_k is a decreasing sequence and that $\alpha_1 = \text{id}$.

Dually, the operator $\beta_k = \text{id} \vee \varepsilon\rho_{10-k}$ is a closing, the *rank-min closing*. The sequence β_k is increasing, and $\beta_1 = \text{id}$.

The proposition stated above implies that the compositions $(\alpha\beta)_k$ and $(\beta\alpha)_k$ are morphological filters. In figure 4 one can see that these filters are eminently suited for noise cleaning.

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